Quiz 3 Solution

Tue. 4:30

Luo Yankeng

① Given $\vec{r}(t) = \langle \frac{1}{3}t^3, \frac{1}{2}t^2, t \rangle$

Find $\vec{T}'(t)$ at $t=1$

$$\vec{r}'(t) = \langle t^2, t, 1 \rangle$$

$$|\vec{r}'(t)| = \sqrt{t^4+t^2+1}$$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \left\langle \frac{t^2}{\sqrt{t^4+t^2+1}}, \frac{t}{\sqrt{t^4+t^2+1}}, \frac{1}{\sqrt{t^4+t^2+1}} \right\rangle$$

$$\vec{T}'(t) = \left\langle \frac{t^3+2t}{(t^4+t^2+1)^{3/2}}, \frac{-t^4+1}{(t^4+t^2+1)^{3/2}}, -\frac{2t^3+t}{(t^4+t^2+1)^{3/2}} \right\rangle$$

$$\vec{T}'(1) = \left\langle \frac{\sqrt{3}}{3}, 0, -\frac{\sqrt{3}}{3} \right\rangle$$

② Given $\vec{r}(0) = \langle 2, 3, 1 \rangle$, $\vec{v}(t) = \langle t^2, t+1, e^{-t} \rangle$

Find $\vec{r}(t)$ for all t.

$$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(s)\,ds$$

$$= \langle 2,3,1 \rangle + \int_0^t \langle s^2, s+1, e^{-s} \rangle\,ds$$

$$= \langle 2,3,1 \rangle + \left\langle \frac{s^3}{3}, \frac{s^2}{2}+s, -e^{-s} \right\rangle \Big|_0^t$$

$$= \langle 2,3,1 \rangle + \left\langle \frac{t^3}{3}, \frac{t^2}{2}+t, -e^{-t}+1 \right\rangle$$

$$= \left\langle \frac{t^3}{3}+2, \frac{t^2}{2}+t+3, -e^{-t}+2 \right\rangle$$