Quiz 4 sol. Luo Yankeng

Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

$$xyz = x + 2y + 3z$$

Sol.

$$\frac{\partial}{\partial x}(xyz) = 1 + 0 + 3\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}(xyz) = y\frac{\partial}{\partial x}(xz) = y\cdot(1\cdot z + x\frac{\partial z}{\partial x})$$

$$\therefore (xy-3)\frac{\partial z}{\partial x} = 1 - yz$$

$$\therefore \frac{\partial z}{\partial x} = \frac{1-yz}{y-3}$$

$$\frac{\partial}{\partial y}(xyz) = 0 + 2 + 3\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial y}(xyz) = x\frac{\partial}{\partial y}(yz) = x\cdot\left(1\cdot z + y\cdot\frac{\partial z}{\partial y}\right)$$

$$\therefore (xy-3)\frac{\partial z}{\partial y} = 2 - xz$$

$$\therefore \frac{\partial z}{\partial y} = \frac{2-xz}{xy-3}$$