Quiz #4.

① Find z_x and z_y if $yz = \log(3x+z)$. Notation $(\ \ )_x = \frac{\partial}{\partial x}$

Solution:

z_x says that z depends on x and you must see y as a constant. Consider the function yz and its partial derivative with respect to x:

$(yz)_x = (y)_x \cdot z + y \cdot z_x$; now y is thought as a constant so $(y)_x = 0$;

thus $(yz)_x = y z_x$.

On the other hand, consider $\log(3x+z)$; this function can be seen like the composition of two functions $\log$ and $3x+z$; this says we need to apply the Chain rule:

$$(\log(3x+z))_x = \frac{1}{3x+z} \cdot (3+z_x).$$

Now as $yz = \log(3x+z)$; then $(yz)_x = (\log(3x+z))_x$; so

$$y z_x = \frac{3+z_x}{3x+z} \Rightarrow (3x+z)y z_x = 3 + z_x \Rightarrow (3x+z)y z_x - z_x = 3.$$

Factorizing z_x we get: $z_x[(3x+z)y - 1] = 3$

$$\Rightarrow \boxed{z_x = \frac{3}{(3x+z)y-1}}$$

Similarly; z_y means z depends on y and x is thought as a constant,

so $(yz)_y = (\log(3x+z))_y \Rightarrow z + y z_y = \frac{1}{3x+z} \cdot z_y$.

$$\Rightarrow z = \frac{z_y}{(3x+z)} - y z_y \Rightarrow z = z_y\left[\frac{1}{3x+z} - y\right] \Rightarrow \boxed{\frac{z}{\left[\frac{1}{3x+z} - y\right]} = z_y.}$$

② Find z_x if $yz = \ln(x+z)$.

Solution: z_x = z depends on x and y is thought as a constant.

① $\frac{d}{dx}(yz) = \frac{d}{dx}(y) \cdot z + \frac{d}{dx}(z) \cdot y = z_x \cdot y$ since $\frac{d}{dx}(y) = 0$ (y constant).

Now:

② $\frac{d}{dx}(\ln(x+z)) = \frac{1}{x+z} \cdot \left[\frac{d}{dx}(x) + \frac{d}{dx} z\right] = \frac{1}{x+z}\left[1 + z_x\right]$; then:

As $\frac{d}{dx}(yz) = \frac{d}{dx}(\ln(x+z))$, we have: from ① and ②.

$$z_x y = \frac{1+z_x}{x+z}, \text{ then } z_x y(x+z) = 1 + z_x$$

- $z_x y(x+z) - z_x = 1$
- $z_x\left[y(x+z) - 1\right] = 1$ (factorizing)
- $\boxed{z_x = \frac{1}{y(x+z)-1}}$