Quiz Solution. 4:30 Luo Yankeng

Show that the line integral $\int_C y^3dx+3xy^2dy$ is independent of path and evaluate the integral if C is any path from (0,0) to (2,4)

proof: If $P(x,y)=y^3$, $Q(x,y)=3xy^2$

$P_y=3y^2$, $Q_x=3y^2$, $P_y=Q_x$

$\therefore \int_C Pdx+Qdy$ is independent of path.

Sol. Suppose, $\nabla f=\langle P, Q\rangle$

then $f_x=y^3$, $f_y=3xy^2$

$\therefore f=xy^3+h(y)$ by integrating f_x

then $f_y=3xy^2+h'(y)$

Compared to $f_y=3xy^2$, $h'(y)=0$, hence $h(y)$ is constant.

$\therefore f(x,y)=xy^3+C$ (C is a content)

$\therefore \int_C y^3dx+3xydy=f(2,4)-f(0,0)$

$=2\times4^3-0$

$=128$