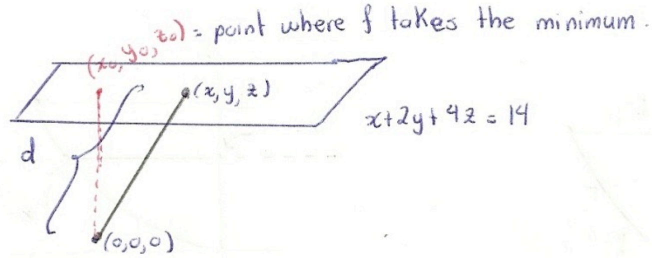
Quiz #6

① Find the point on the plane closest to origin:

$$x + 2y + 4z = 14.$$

Solution



This is a problem of distance; now the distance from $(0,0,0)$ to any point (x, y, z) is given by $d(x,y,z) = \sqrt{x^2 + y^2 + z^2}$. We want to minimize this function under the condition $x + 2y + 4z = 14$, but as the function $\sqrt{\cdot}$ is increasing; it suffices to minimize $x^2 + y^2 + z^2$. Consider

$$\begin{cases} f(x,y,z) = x^2 + y^2 + z^2 \\ g(x,y,z) = x + 2y + 4z - 14 = 0. \end{cases}$$ and the equation $\nabla f = \lambda \nabla g$;

then we get: $\nabla f(x,y,z) = (2x, 2y, 2z)$ and $\nabla g(x,y,z) = (1,2,4)$; so $\nabla f = \lambda \nabla g$ implies: $(2x, 2y, 2z) = \lambda(1,2,4)$; therefore:

$$(1) \begin{cases} 2x = \lambda \\ 2y = 2\lambda \\ 2z = 4\lambda \end{cases} \Rightarrow (2) \begin{cases} 2x = \lambda \\ y = \lambda \\ z = 2\lambda. \end{cases}$$ Now using the equation:

$x + 2y + 4z = 14$ we have: $\frac{\lambda}{2} + 2\lambda + 8\lambda = 14 \Rightarrow \lambda + 4\lambda + 16\lambda = 28$;

Then: $21\lambda = 28$; so $\lambda = \frac{28}{21}$ and we conclude from (2) that:

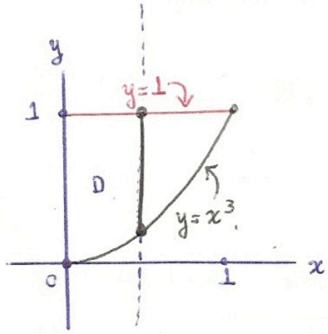
$x = \frac{28}{42}$; $y = \frac{28}{21}$ and $z = \frac{56}{21}$. So the closest point is $\left(\frac{28}{42}, \frac{28}{21}, \frac{56}{21}\right)$

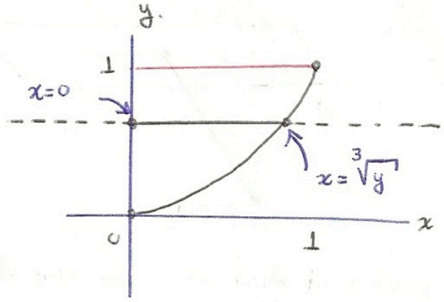
$$\parallel$$

$$\left(\frac{2}{3}, \frac{4}{3}, \frac{8}{3}\right).$$

(2) Reverse order of $\int_0^1 \int_{x^3}^1 4\pi \sin(xy^{\frac{4}{3}})\, dy\, dx. = I.$

Solution:





The integral I is telling us that $x \in [0,1]$ and that we must picture the functions $y = x^3$ and $y = 1$ when $x \in [0,1]$. We are integrating over D. Now when we reverse order we see in what interval is y; in this case $y \in [0,1]$; and now x must be seen as function of y, what we do is to picture a parallel line to the x-axis; intersecting the domain D. and we see this line intersects the boundary of D when $x = 0$ and $x = \sqrt[3]{y}$; so our new integral is:

$$I = \int_0^1 \int_0^{\sqrt[3]{y}} 4\pi \sin(xy^{\frac{4}{3}})\, dx\, dy.$$