

Rec 2:30 3:30

1. Let c be the parabola joining $(-2, 4)$ through $(0, 0)$ to $(2, 4)$
a portion of the parabola $y = x^2$.

Find $\int_c x^2 y dy$

Solution: $\int_c x^2 y dy = \int_{-2}^2 x^2 (x^2) 2x dx = \int_{-2}^2 2x^5 dx = 0$

2. Consider the region bounded below by the cone $z^2 = x^2 + y^2$
and above by the sphere $x^2 + y^2 + z^2 = 20$.

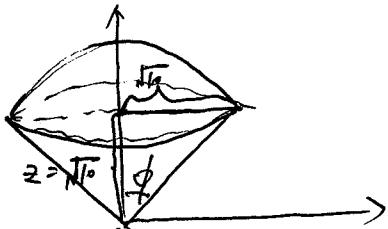
a) Where do these surfaces intersect?

b) set up an integral, including the limits which give
this volume?

Solution:

a)
$$\begin{cases} x^2 + y^2 + z^2 = 20 \\ z^2 = x^2 + y^2 \end{cases} \Rightarrow 2z^2 = 20 \Rightarrow z = \sqrt{10}$$

b)
$$\iiint dV = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{20}} \rho^2 \sin \phi d\rho d\phi d\theta$$



$\tan \max \phi = \frac{\sqrt{10}}{\sqrt{10}} = 1 \Rightarrow \max \phi = \frac{\pi}{4}$

$\rho^2 = 20 \Rightarrow \rho = \sqrt{20}$