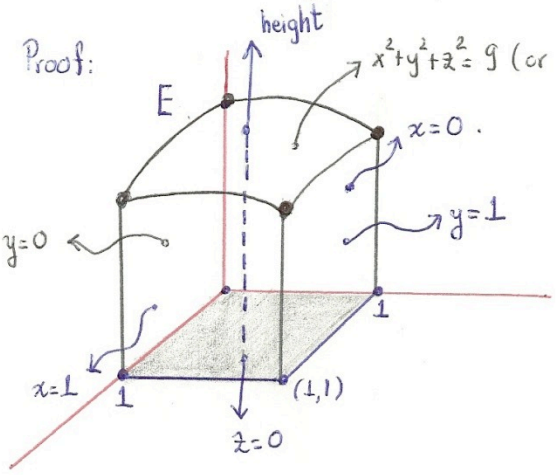
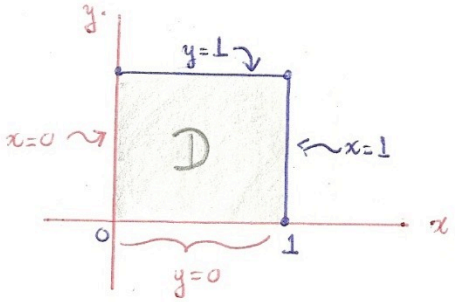
Quiz # 7.

Evaluate the integral $\iiint_E z\,dV$ where E is the solid bounded above by the sphere $x^2+y^2+z^2=9$, below by the plane $z=0$, and on the sides by the planes $x=0$, $x=1$, $y=0$, and $y=1$.

Proof:





Observe that the solid E is determined by the shadowed region D; then the triple-integral is given by:

$$I=\iiint_E z\,dV=\underbrace{\int_0^1\int_0^1}_{\text{describing "the base" of the solid E}}\ \underbrace{\int_0^{\sqrt{9-x^2-y^2}}}_{\text{describing the height}} z\,dz\,dy\,dx.$$

$$=\int_0^1\int_0^1 \frac{z^2}{2}\Big|_{z=0}^{z=\sqrt{9-x^2-y^2}}\,dy\,dx=\frac{1}{2}\int_0^1\int_0^1 (9-x^2-y^2)\,dy\,dx$$

$$=\frac{1}{2}\int_0^1 \left(9y-x^2y-\frac{y^3}{3}\right)\Big|_{y=0}^{y=1}dx=\frac{1}{2}\int_0^1\left(9-x^2-\frac{1}{3}\right)dx$$

$$=\frac{1}{2}\left[9x-\frac{x^3}{3}-\frac{1}{3}\cdot x\Big|_{x=0}^{x=1}\right]=\frac{1}{2}\left[9-\frac{1}{3}-\frac{1}{3}\right]=\frac{25}{6}.$$