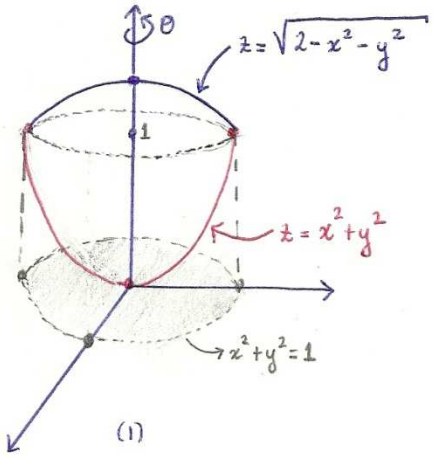
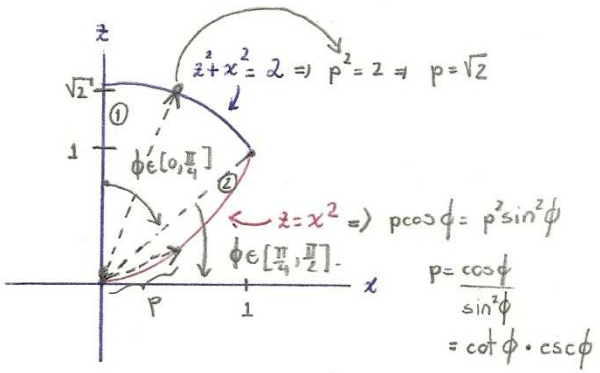
Quiz # 8.

1) Consider the solid E bounded above by $x^2+y^2+z^2=2$ and below by $z=x^2+y^2$. Express the volume of E in spherical coordenates.

Solution:



(1)



(2)

Notice the sphere and paraboloid intersect: $\begin{cases} x^2+y^2+z^2=2 \\ z=x^2+y^2 \end{cases}$ when $z^2+z=2$

$z^2+z-2=0$; $(z-1)(z+2)=0$; so $\underline{z=1}$ or $z=-2$

only possibility since $z\geq 0$

Notice the solid E is obtained by rotating picture (2) around the z-axis; so $0\leq\theta\leq 2\pi$. And when $\theta=0$ we obtain the portion (2) and $\begin{cases} x=\rho\sin\phi \\ y=0 \\ z=\rho\cos\phi \end{cases}$ (since $\cos 0=1$ and $\sin 0=0$). (Remember ϕ is measure from z-axis).

So $\text{Volume}(E)=\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{2}}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta+\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{\frac{\cos\phi}{\sin^2\phi}}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$