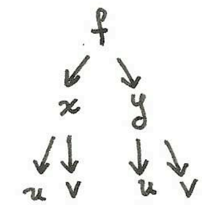
Quiz # 5

① a) Let $g(u,v) = f(\sin(u)e^v,\ u^2 - v^2 - u)$. Find g_u.

Set $x = \sin(u)e^v$ and $y = u^2 - v^2 - u$



Then

$$g_u(u,v) = f_x(x,y) \cdot x_u + f_y(x,y) \cdot y_u .$$

$$= f_x(x,y)\, e^v \cdot \cos(u) + f_y(x,y) \cdot (2u - 1).$$

b) Suppose $g_u(0,1) = 1$ and $f_y(0,-1) = 0$. Find $f_x(0,-1)$.

Notice when $u=0$ and $v=1$ we get that $x = \sin(0)e^1 = 0$ and $y = 0^2 - (1)^2 - 0 = -1$. Then:

$$g_u(0,1) = f_x(0,-1)\, e^1 \cos(0) + f_y(0,-1)\,(2 \cdot 0 - 1)$$

so $\quad 1 = f_x(0,-1) \cdot e$; then $f_x(0,-1) = \frac{1}{e} = e^{-1}$.

② Find the tangent plane at $(1,1,e) = (x_0, y_0, z_0)$ if $z = e^{xy} = f(x,y)$.

We Know the formula for tangent plane for functions of two variables is given by: $z - z_0 = f_x(x_0,y_0)(x - x_0) + f_y(x_0,y_0)(y - y_0)$
Then the tangent plane is:

$$z - e = f_x(1,1)(x-1) + f_y(1,1)(y-1)$$. Now $f_x = y e^{xy}$

and $f_y = x e^{xy}$; so $z - e = e(x-1) + e(y-1)$.

We may use $F(x,y,z) = z - e^{xy} = 0$ (level surface) and the tangent plane is given by: $\nabla F(x,y,z) = (-ye^{xy}, -xe^{xy}, 1)$

$$\nabla F(1,1,e) \cdot (x-1, y-1, z-e) = 0 \quad \therefore \quad (-e, -e, 1) \cdot (x-1, y-1, z-e) = 0$$