7 Wednesday, January 28

Reminder: Exam 1 is on Monday, February 9, 8:00pm-9:00pm in Elliott Hall. Next Friday will be a review day, and the following Monday will be an optional review day.

First-Order Linear Differential Equations

Theorem 7.1. *An integrating factor for the first-order linear differential equation*

$$y' + P(x)y = Q(x)$$

is $\mu(x) = e^{\int P(x)\,dx}$ *(any antiderivative will do). The solution of the differential equation is*

$$y = \frac{1}{\mu(x)}\left[\int Q(x)\mu(x)\,dx + C\right] = \frac{1}{\mu(x)}\int Q(x)\mu(x)\,dx + \frac{C}{\mu(x)} \tag{7.1}$$

Solving first-order linear differential equations:

(1) Put the equation in standard form.

(2) Compute an antiderivative of $P(x)$. Any antiderivative will do, but it will be easiest to just use a zero arbitrary constant.

(3) Find $\mu(x) = e^{\int P(x)\,dx}$.

(4) Use Eq. (**??**) to solve for y.

Example 7.2. Solve the differential equation.

(1) $y' = \frac{x^3 - 2y}{x}$

(2) $y' + y = \frac{1}{1+e^x}$

(3) $(2y+1)\,dx + \left(\frac{x^2-y}{x}\right)dy = 0$

(4) $y' \sin x + y \cos x = e^x$

(5) $y' + 2xy = 2x$

Example 7.3.

(1) A tank wil a capacity of 500 gal originally contains 200 gal of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow.

(2) A ball with mass 0.1 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m high.

(a) Neglecting air resistance, find the maximum height above the ground that the ball reaches.

(b) Assuming the ball misses the building on the way down, find the velocity of the ball when it hits the ground.

(c) Now assume there is a force due to air resistance of $-v/30$. What is the maximum height above the ground that the ball reaches?