MA514: COMPUTER PROJECT #1
FALL 2009

DUE DATE: OCTOBER 1ST, THURSDAY

Note:. Please follow the guidelines:

- Clearly label all plots.
- Use *subplot* command to arrange figures and keep the total number of pages low.
- Add a comment on each function script to describe its purpose and also include your name.
- You may use another programming language, so long as the same requirements of each problem are achieved.

1. SOURCE CODE:. Let $x_0, \ldots, x_n$ be a set of distinct points with corresponding data values $f_0, \ldots, f_n$. Let $p(x)$ be the unique polynomial that interpolates the data such that $p(x_i) = f_i$, $i = 0, \ldots, n$.

Write the following MATLAB routines:

1. *F=DivDiff(x,f,n)* — a function the compute the divided differences.
2. *p=Horners(x,F,xbar)* — a function that uses the divided differences to evaluate $p(x)$ at a given point *xbar* with the Horners' algorithm.

2. Runge Function. Consider the function

$$f(x) = \frac{1}{1 + 25x^2}.$$

(a) Let x_i be 11 equally spaced points in $[-1, 1]$. That is,

$$x_i = -1 + i/5, \qquad i = 0, \ldots, 10.$$

1. Use your code from the previous problem to construct the divided differences for these x_i's.
2. Use your Horner's method code to evaluate the interpolation polynomial p at 500 equally spaces points $z_k, k = 1, \ldots, 500$, in $[-1, 1]$.
3. Make a single plot that contains: $(z, p(z))$, $(z, f(z))$, and the data points $(x_i, f(x_i)), i = 0, \ldots, 10$. Use solid line for $p(z)$, dashed line for $f(z)$, and stars (no line) for the data points.

(b) Now let x_i be 11 unequally spaced points in $[-1, 1]$, defined by,

$$x_i = \cos\left[\frac{\pi(i + 0.5)}{11}\right], \qquad i = 0, \ldots, 10.$$

Repeat the same steps in part (a).

(c) Explain what you observe with these two choices of points.