Theorem. Let $f, g$ be real polynomials, and suppose that their Wronski determinant $W(f, g)=f^{\prime} g-f g^{\prime}$ has all zeros real. Let $I$ be a real interval containing no zeros of $W$. Then any linear combination $a f+b g$ has at most one root on $I$.

Can this be generalized to more than 2 polynomials? The simplest unsolved case is

Conjecture. Let $f, g, h$ be real polynomials, and suppose that their Wronskian $W(f, g, h)$ has all zeros real. If $I$ is a real interval containing no zeros of $W$ then any linear combination $a f+b g+c h$ has at most two roots on $I$.

