Equilibrium points of logarithmic and Newtonian potentials

For an infinite discrete set of point masses in space, is it always true that the force created by these masses vanishes at some point?

The gravity force created by masses $a_k > 0$ placed at the points x_k in $\mathbf{R}^n$ is

$$F(x) = \sum_{k=0}^{\infty} \frac{a_k(x_k - x)}{|x - x_k|^n},$$

which converges when

$$\sum_{k=0}^{\infty} \frac{a_k}{|x_k|^{n-1}} < \infty. \tag{1}$$

The question is whether every such F has at least one zero.

If we consider potential kernel,

$$K(x) = \begin{cases} \log|x| & \text{for } n = 2, \\ -|x|^{2-n} & \text{for } n \geq 3 \end{cases}$$

then the potential

$$u(x) = \sum_{k=0}^{\infty} a_k K(x - x_k)$$

converges if

$$\sum_{k=0}^{\infty} \frac{a_k}{|x_k|^{n-2}} < \infty, \tag{2}$$

which is a stronger condition than (1). So in the general case the potential has to be defined by

$$u(x) = \sum_{k=1}^{\infty} a_k(K(x - x_k) - K(x_k)),$$

which converges under the condition (1). In any case we have $F(x) = \text{const}\nabla u(x)$, so the equilibrium points coincide with the critical points of the potential.

Existence of equilibrium points is known only in the following two cases:
(i) $n \geq 2$, (2) holds, and $a_k > a$ with some $a > 0$ [1], or
(ii) $n = 2$, (1) holds, and $a_k > a$ with some $a > 0$ [2].

In the case $n = 2$ one uses complex variables, namely the fact that the complex conjugate $\overline{F(x)}$ is a meromorphic function, which permits to obtain a stronger result.

References

[1] J. Clunie, A. Eremenko and J. Rossi, On equilibrium points of logarithmic and Newtonian potential, Journal of the London Math. Soc., 47, 1993, 309-320.

[2] A. Eremenko, J. Langley and J. Rossi, On the zeros of meromorphic functions of the form $\sum_{k=1}^{\infty} \frac{a_k}{z-z_k}$. Journ. d'Analyse Math., 62 (1994), 271-286.