

# Equilibrium points of Newtonian potentials

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Consider the potential generated by finitely many positive charges in 3-space:

$$u(x) = \sum_{k=1}^n \frac{a_k}{|x - x_k|}, \quad a_k > 0.$$

A point  $x$  is an equilibrium point if  $\text{grad } u(x) = 0$ .

Question 1. Is the set of equilibrium points always finite? (It can be infinite if we allow charges  $a_k$  of different signs).

Question 2. If the set of equilibrium points is finite, how many points can it contain?

The answer to question 2 is unknown even if there are three points and all  $a_k = 1$ . J. C. Maxwell stated without proof that the number of equilibrium points is at most  $(n - 1)^2$ . This gives 4 for  $n = 3$  which would be best possible. Gabrielov, Novikov and Shapiro proved that for  $n = 3$  there cannot be more than 12 equilibrium points.

## References

- [1] A. Gabrielov, D. Novikov and B. Shapiro, The mystery of point charges, arXiv:math-ph/0409009
- [2] J. C. Maxwell, Treatise on electricity and magnetism, vol. 1, Dover, 1954.