1. The linearization of a non-linear system near a critical point has the form

\[ u' = Au, \quad A = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}. \]

Based on this linearization can we say that the critical point of the non-linear system is

A. Center
B. Focus
C. Saddle
D. Stable node
E. Cannot be determined.
2. The point \((1, 0)\) is a critical point of the system

\[
\begin{align*}
x' &= -x + x^2 + y^2 \\
y' &= y + xy.
\end{align*}
\]

The linearized system

\[u' = Au\]

has the matrix

A. \[
\begin{pmatrix}
1 & 1 \\
4 & 1
\end{pmatrix}
\]

B. \[
\begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}
\]

C. \[
\begin{pmatrix}
1 & 0 \\
0 & 2
\end{pmatrix}
\]

D. \[
\begin{pmatrix}
1 & 1 \\
-4 & 1
\end{pmatrix}
\]

E. \[
\begin{pmatrix}
1 & 1 \\
4 & -1
\end{pmatrix}
\]
3. The populations $x(t)$ and $y(t)$ of two species are governed by the system

$$
\begin{align*}
x' &= x(2 - 2x - y) \\
y' &= y(2 - x - 2y).
\end{align*}
$$

Let $(x(t), y(t))$ be the solution with initial conditions $(x(0), y(0)) = (1/2, 3/4)$. As $t \to +\infty$, the behavior of this solution can be described as

A. The two species coexist, and the solution is periodic.
B. Population $x$ will become extinct but population $y$ will survive.
C. Population $y$ will become extinct but population $x$ will survive.
D. The two species will coexist and the solution converges to an equilibrium point $(x_0, y_0)$ with both $x_0, y_0$ positive.
E. Both species will become extinct.
4. The populations $x(t)$ and $y(t)$ of two species are governed by the system

\[
\begin{align*}
x' &= x(2 - y) \\
y' &= y(-4 + x).
\end{align*}
\]

Let $(x(t), y(t))$ be the solution with initial conditions $(x(0), y(0)) = (1/2, 3/4)$. As $t \to +\infty$, the behavior of this solution can be described as

A. The two species coexist, and the solution is periodic.
B. Population $x$ will become extinct but population $y$ will survive.
C. Population $y$ will become extinct but population $x$ will survive.
D. The two species will coexist and the solution converges to an equilibrium point $(x_0, y_0)$ with both $x_0, y_0$ positive.
E. Both species will become extinct.
5. The eigenvalues of the boundary value problem

\[ y'' = \lambda y, \quad y(0) = 0, \quad y'(\pi) = 0 \]

are

A. \( \lambda_n = -(n - 1/2)^2, \quad n = 1, 2, 3, \ldots \)
B. \( \lambda_n = -(n - 1/2)^2\pi^2, \quad n = 1, 2, 3, \ldots \)
C. \( \lambda_n = -n^2, \quad n = 1, 2, 3, \ldots \)
D. \( \lambda_n = -n^2, \quad n = 0, 1, 2, 3, \ldots \)
E. None of the above.
6. Consider the boundary value problem

\[ y'' + y = e^x, \quad y(0) = 0, \quad y(\pi) = 0. \]

The following is true:

A. The solution is \( y(x) = e^x / 2 \).
B. The solution is \( y(x) = (e^x - \cos x)/2 \).
C. The solution is \( y(x) = C_1 \cos x + C_2 \sin x + e^x / 2 \).
D. The solution is \( y(x) = \sin x \).
E. There is no solution.
7. The system
\[ x' = y^2 - y, \quad y' = 2y + xy \]
has the trajectories described by the following equation

A. \( y^2 - y = 2y + xy + C \)
B. \( y^2 - y = 2x + x^2/2 + C \)
C. \( y^2/2 - y = 2x + x^2/2 + C \)
D. \( y^2/2 - y = 2y + xy + C \)
E. \( y^2 - y = 2y + xy. \)
8. Consider a uniform rod of length 50 cm with ends kept at temperature 0°, made of material with thermal diffusivity 1. Assume that the initial temperature is

\[ f(x) = \sin(\pi x/10). \]

Then the temperature \( u(x, t) \) is given by

A. \[ u(x, t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-n^2 \pi^2 t/100} \sin \frac{\pi n x}{50}. \]

B. \[ u(x, t) = e^{-\pi^2 t/100} \sin \frac{\pi x}{10}. \]

C. \[ u(x, t) = e^{-\pi^2 t/10} \sin \frac{\pi x}{50}. \]

D. \[ u(x, t) = \sum_{n=1}^{\infty} \frac{1}{n} e^{-n^2 \pi^2 t/100} \sin \frac{\pi n x}{50}. \]

E. \[ u(x, t) = \sum_{n \text{ odd}} \frac{1}{n} e^{-n^2 \pi^2 t/100} \sin \frac{\pi n x}{10}. \]
9. Let \( \tilde{f} \) be the odd 2-periodic extension of the function \( f(x) = x \) on \((0, 1)\).
What is \( \tilde{f}(13.3) \)?

A. 1.3
B. 13.3
C. \(-0.7\)
D. 0.7
E. None of the above.
10. Consider a uniform rod of length 50 cm with left end kept at temperature 
10° and the right end kept at the temperature 35°. Assume that the initial 
temperature is given by 

\[ f(x) = \cos(\pi x/10). \]

Then as \( t \to +\infty \) the temperature \( u(15, t) \) at the point at 15 cm from the 
left end tends to

A. 7.5°  
B. 10°  
C. 17.5°  
D. 22.5°  
E. 25°.