Practice problems for exam 2

1. Find the real general solution of the equation
   \[ y'' + y' = \cos t + t. \]

2. Find the real general solution of the equation
   \[ y'' + y = \sin t + t^2. \]

3. Find the general solution of the equation
   \[ y'' + 3y' + 2y = \frac{1}{e^x + 1}. \]

4. Find the general solution of the equation
   \[ y''' - 6y'' + 9y' = xe^{3x} + e^{3x} \cos(2x). \]

5. Are these functions linearly independent?
   \[ 2t - 3, \quad t^3 + 1, \quad 2t^2 - t, \quad t^2 + t + 1. \]

6. For which \( \omega \) does this equation have at least one solution which is
   unbounded when \( t \to +\infty \)?
   \[ y'' + 4y = \sin \omega t. \]

7. Factor the the following polynomials into factors of degree 1:
   \[ \lambda^3 - \lambda^2 + 2\lambda - 2, \]
   \[ \lambda^3 - 1, \]
   \[ \lambda^3 + 1, \]
   \[ \lambda^5 - 1, \]
   \[ \lambda^5 + 1. \]