

Zeros of sums of simple fractions

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Let $z_n \rightarrow \infty$ be a sequence in the complex plane. Consider a meromorphic function

$$f(z) = \sum_{n=1}^{\infty} \frac{a_n}{z - z_n}, \quad \sum_{n=1}^{\infty} |a_n| < \infty, \quad \sum_{n=1}^{\infty} a_n \neq 0.$$

Is it true that each such f must have zeros?

This is known under various additional conditions, for example

- a) f is of finite order,
- b) $a_n > 0$.