Accessory parameter of the Heun equation

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Consider the Heun equation

\[ y'' + \left( \sum_{j=0}^{2} \frac{1 - \alpha_j}{z - a_j} \right) y' + \frac{A z - \lambda}{(z - a_0)(z - a_1)(z - a_2)} y = 0, \]

where the parameters \( \alpha_j, a_j, A \) satisfy \( \alpha_j > 0 \),

\[ A = \alpha' \alpha'', \quad \sum_{j=0}^{2} \alpha_j + \alpha' + \alpha'' = 2, \]

where \( \alpha' \) and \( \alpha'' \) are real. \( \lambda \) is called the accessory parameter.

Problem. For given \( a_j, \alpha_j, A \), describe the set of values of \( \lambda \) for which the monodromy group of the equation is conjugate to a subgroup of \( SU(2) \).

Same question when the parameters \( a_j \) and \( \lambda \) are also real.

For which parameters \( a_j, \alpha_j, A \) is this set non-empty? Is it always finite? How many elements can it contain?