

Homework 9

1. a) For which positive integers m is it possible that 2^m and 2^{m+1} have equal sums of digits in decimal system? *Hint:* Every number has the same residue modulo 3, as sum of its digits in decimal system. So, for example 721, 020, 013, 214 has residue 2 modulo 3.

b) How to find out quickly, whether a given integer is divisible by 11?

2. a) Express a primitive root of 1 of degree 8 in the form $a + bi$ (your expression may contain radicals but no sines, cosines or exponentials).

b*) Do the same for a primitive root of degree 5.

3. a) A complex number w is called primitive root of 1 of degree d , if all roots of 1 of degree d are powers of w . Which roots of 1 of degree 12 are primitive, and which are not?

b*) Is $\exp 7\pi i/60$ a root of 1 of degree 120? If yes, is it primitive?

Definition. Fourier matrix of size $N \times N$ is the matrix with elements $a_{i,j} = w^{-ij}$, where $w = \exp 2\pi/N$ and $i, j = 0, \dots, N - 1$. 4. Write explicitly the Fourier matrix 6×6 .

5. Let A be the 4×4 Fourier matrix. Find A^2 and A^4

6. a) Show that the 4-th power of every Fourier matrix is a multiple of the unit matrix.

b) How does the second power of a Fourier matrix look?

7. Find the Fourier transform of the vector $(2, 1, -2, 1)$.

8. According to Danielson and Lanczos, their first calculation at Purdue, took 10 min to find the FT of a 8-vector (all by hand, of course), 25 min for a 16-vector, 60 min for a 32-vector, and 140 min for a 64-vector. Plot these data, and discuss, whether are consistent with the theoretical result that the time is proportional to $N \log N$. Estimate the coefficient of proportionality. (Please do all calculations by hand:-)

9. Prove Parseval's identity:

$$\sum_{n=0}^{N-1} |F(n)|^2 = N \sum_{n=0}^{N-1} |f(n)|^2.$$

10. Suppose that a vector (a, b, c, d, e, f, g, h) has FT (A, B, C, D, E, F, G, H) . Find the vector (of dimension 4), whose FT is (A, C, E, G) .