Practice problems for the text

1. A plant damps radioactive vaste in the form of water solution containing $a\ kg/m^3$ of the radioactive substance with half-life $T sec$. This solution flows into a basin of volume V at the rate of $b\ m^3/sec$. The excess from the basin is damped to a nearby river at the same rate $b\ m^3/sec$.

Suppose that this system was operated for long time with the same constant parameters. What is the concentration of the substance in the basin? Can the concentration of the solution that flows to the river made arbitrarily small by increasing the volume of the basin?

2. Consider two tanks A and B of the volume 100 liters each. Initially tank A contains 1 kg of a chemical dissolved in 100 liters of water, and tank B is filled with 100 liters of clean water.

At some time, clean water begins to flow into tank A at the rate of 20 liters per minute, and after good mixing, the excess solutiuon goes to tank B with the same rate. The overflow is removed from tank B, again at the same rate. (Dont ask me where does it go. I suspect that it goes again to a nearby river).

At what time the amount of the chemical in tank B will be maximal?

3. Write a differential equation for which every function $y(x) = cx^m$ is a solution (for arbitrary constant c and non-negative integer m).

4. Find as 1-st order differential equation, whose general solution is

$$x^2y + e^{-y} = C.$$

5. Find the equilibrium solutions for $x' = x^3 - 2x^2 - 4x + 8$ and tell which of them are stable.

6. Find the general solution of $ydx = (\sqrt{x^2 + y^2} + x)dy$.

7. Find the general solution of $dy/dx = (x + y + 2)/(x - y + 1)$. (Hint: find a simple change of the variables that makes it homogeneous.)

8. Solve the following initial value problems:

$$y' + 2y - x = 0, \quad y(0) = 0,$$
$$y' + 2xy = x^3, \quad y(0) = 0,$$

If you want more challenging problems, look at
http://www.math.purdue.edu/ eremenko/airplane.html, or
http://www.math.purdue.edu/ eremenko/jo.html