1. Let \( y(t) \) be the amount of the substance in the pool, in kilograms. Then

\[
y'(t) = ab - ky - by/V,
\]

Here the first summand reflects pumping into the pool, second term reflects the radioactive decay, and the third term is for overflow from the pool to the river. The constant \( k \) is related to the half-life by the formula \( k = \log 2/T \) (see the book). This is a separable equation, but the problem only asks for the equilibrium. Equilibrium happens when

\[
y = \frac{ab}{\log 2/T + b/V}.
\]

This is the amount of the substance in the equilibrium. For concentration, one has to divide by \( V \). We obtain

\[
\frac{ab}{V \log 2/T + b}.
\]

So by making large enough pool we can decrease the concentration to any given amount. Especially when the half-life is not too long.

2. Let \( x(t) \) be the amount of the chemical in the tank \( A \), and \( y(t) \) in the tank \( B \). We have

\[
x' = -20x/100, \quad y(0) = 1,
\]

and

\[
y' = 20x/100 - 20y/100, \quad y(0) = 0.
\]

Solving the first equation we get \( x(t) = e^{-t/5} \). Now the second one becomes a linear first order equation

\[
y' = e^{-t/5}/5 - y/5.
\]

Solving with the initial condition \( y(0) = 0 \), we obtain

\[
y(t) = \frac{1}{5}te^{-t/5}.
\]

The maximum is at \( t = 5 \), and we have \( e^{-1} \) kg of the chemical at this moment.
3, 4 were solved in class.

5. The right hand side factors as $(x - 2)^2(x + 2)$, so 2 is unstable and -2 is unstable but semistable.

6. This is homogeneous. Setting $x = wy$ ($y$ is the independent variable) we get $w' y = \sqrt{w^2 + 1}$. This is separable

$$\frac{dw}{\sqrt{w^2 + 1}} = \frac{dy}{y},$$

$$\sinh^{-1} w = \log |y| + C.$$

$$w = \sinh(\log |y| + C), \quad x = y \sinh(\log |y| + C).$$

7. This is neither homogeneous nor exact but can be converted into a homogeneous one by introducing new variables $u, v$ by the formulas

$$y = u - 1/2, \quad x = v - 3/2.$$

so that Then for $u$ and $v$ we have a homogeneous equation

$$\frac{du}{dv} = -\frac{u + v}{v - u}.$$ Such equation we solved in class. After solving this we return to $x, y$ by the above formulas.

8. This is exact.

$$(x + y + 2)dx + (1 + x - y)dy = 0 = Mdx + Ndy.$$ $M_y = 1 = N_x$. The implicit solution is

$$\frac{1}{2} x^2 - \frac{1}{2} y^2 + xy + 2x + 2y + C = 0.$$

9. This equation does not belong to any class we studied, and finding a general solution is difficult. However, it is only asked for a solution of the initial value problem, and this has an evident solution $y(x) \equiv 0$. Existence and uniqueness theorem guarantees that there are no other solutions.

10. 

$$-\frac{1}{2} t \cos t + c_1 \cos t + c_2 \sin t.$$