Progress in Entire and Meromorphic Functions

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1.6 It is proved in [14] that for every countable set E in the complex plane and every $\lambda > 1/2$ there exists an entire function of order λ, whose set of deficient values coincides with E. This completely answers the first question. For the second question a counterexample is constructed in [16]: for every $\lambda > 1/2$ and for every sequence of complex numbers (a_k) there is an entire function f of order λ with the property $\delta(a_k, f) > c^k,\ k = 1, 2 \ldots,$ or some $c \in (0, 1)$. On the other hand, Lewis and Wu [41] proved $\sum \delta^\alpha(a_k, f) < \infty$ for entire functions of finite order with an absolute constant $\alpha < 1/3 - 2^{-264}$.

1.16 Miles [47] gave a positive answer, by showing that for every meromorphic function

$$\liminf_{r\to\infty} \frac{\max_a n(r, a)}{A(r)} \leq e - 10^{-28}.$$

1.18 The last case which remained unsolved, $l = 2$, was completely settled by Langley [34], who proved that the only meromorphic functions f for which ff'' is zero-free, are $f(z) = \exp(az + b)$ and $f(z) = (az + b)^{-n}$. An earlier paper on the subject, [20] contains a gap in the case $l = 2$.

1.19 The last case which remained unsolved, $n = 1$, was settled in [6]: for every non-constant meromorphic function f, the equation $f'(z)f(z) = c$

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has solutions for every $c \neq 0, \infty$. This was first proved by Bergweiler and Eremenko for functions of finite order; then these authors [6], Chen and Fang [11] and Zalcman [66] independently noticed that a general result of Pang [49] permits an extension to arbitrary meromorphic functions. The proof actually applies to all $n \geq 1$.

1.20 This question is equivalent to 1.19. The last remaining case $n = 3$, which corresponds to $n = 1$ in 1.19, was solved in [6].

1.33 This is a slightly more precise conjecture than 2.25. Both problems are solved completely by the following theorem [17]: Suppose f is a meromorphic function of lower order $\lambda < \infty$, and

$$N_1(r, f) := N(r, 1/f') + 2N(r, f) - N(r, f') = o(T(r, f)).$$

Then:
a) 2λ is an integer greater or equal than 2.
b) $T(r, f) = r^\lambda \ell(r)$, where ℓ is a slowly varying function in the sense of Karamata,
c) $\sum \delta(a, f) = 2$, all deficient values are asymptotic, and all deficiencies are multiples of $1/\lambda$.

1.35 Steinmetz [61, 62] proved that every meromorphic solution of a homogeneous algebraic differential equation of second order has the form $f = (g_1/g_2)\exp(g_3)$, where g_i are entire functions of finite order, thus $T(r, f) = O(\exp(r^k))$, for some $k > 0$. Due to work of Wiman and Valiron (see, for example [64]) it is known that "most" algebraic differential equations do not have entire solutions of infinite order. A precise statement of this sort is contained in Hayman's paper [30].

1.37 We mention a result [56]: on every open Riemann surface there exists a holomorphic function with prescribed divisors of zeros and critical points, subject to the trivial restrictions.

1.42 This was completely solved by by Brüggemann [9] who proved the following. Let a linear differential operator

$$L(f) = f^{(n)} + \sum_{j=0}^{n-2} a_j f^j$$

with polynomial coefficients a_j be given, with at least one nonconstant a_j. Then the only meromorphic functions f with infinitely many poles, satisfying $fL(f) \neq 0$, are of the form $f = (H')^{-(n-1)/2} H^{-l}$, where l is a natural integer, and H''/H' is a polynomial.

2.6 The following is proved in [13]. Let f be an entire function of order λ and lower order μ. Then there exists an asymptotic path Γ, such that

$$\log |f(z)| > (A(\lambda, \mu) + o(1)) \log |z|, \quad z \to \infty, \ z \in \Gamma,$$

where $A(\lambda, \mu)$ is some explicitly written function with the property $A(\lambda, \mu) > 0$ for $0 < \mu < \lambda < \infty$.

2.9 If $f(z) \neq 0$, then for every K the level set $|f(z)| = K$ contains a curve tending to infinity. Under this condition Rossi and Weitsman [55] proved that there is an asymptotic curve Γ with the following properties:

$$\log |f(z)| > |z|^{1/2-\epsilon(z)}, \quad \text{where } \epsilon(z) \to 0, \quad \text{and} \tag{1}$$

$$\int_\Gamma (\log |f|)^{-(2+\alpha)} |dz| < \infty \quad \text{for all } \alpha > 0. \tag{2}$$

On the other hand, Barth, Brannan and Hayman [2] constructed a zero-free entire function, having no asymptotic curve, satisfying (1) with $\epsilon = 0$. Furthermore, Brannan pointed out that for their example every asymptotic curve satisfies

$$\int_\Gamma (\log |f(z)|)^{-2} |dz| = \infty.$$

2.11 Nazarov [48] proved that each of the following conditions
a) $\lambda_{k+1} + \lambda_{k-1} \geq 2\lambda_k$ and $\sum(1/\lambda_k) < \infty$, or
b) $\sum(\log \log k)/\lambda_k < \infty$
implies

$$\limsup_{z \to \infty, z \in \Gamma} \frac{\log |f(z)|}{\log M(|z|, f)} = 1.$$

for every curve Γ, tending to infinity.

2.12a Fryntov [22] proved the following partial result. Suppose f is an entire function of lower order μ with density of non-zero exponents Δ. If $\lambda\Delta < 1/3$, and γ is a curve which intersects each circle $|z| = r$ at most once, then

$$\limsup_{z\to\infty, z\in\gamma} \frac{\log|f(z)|}{\log M(|z|)} \geq 2\cos\pi\lambda\Delta - 1.$$

2.19 Let f be entire, of exponential type σ, $|f(z)| \leq A$ for $z < 0$ and $|f(z)| < B$ for $z > 0$. A sharp bound for $|f(z)|$, $z \in \mathbf{R}$ was found in [15]. For complex z nothing is known, except in the case $A = B$.

2.20 This was proved by Bergweiler [3]: if f is a transcendental entire function, and $n \geq 2$, then f has infinitely many periodic points of exact period n. This also follows from the result in [4]

2.21 The existence of repulsive fixed points was proved for the first time by Baker [1], who used Ahlfors' theory of covering surfaces. Since then the proof of this important result was generalized to meromorphic functions and ultimately evolved into an elementary half-page argument of Berteloot and Duval [8].

2.23 Bergweiler, Clunie and Langley [5] proved the conjecture by showing that for every transcendental entire function f and every line, infinitely many of the fixed points of every n-th iterate, $n \geq 2$ do not lie on this line. Later Bergweiler [4] improved this by showing that for every line there are infinitely many repelling fixed points of each n-th iterate, $n \geq 2$, which do not lie on this line.

2.25 The negative answer follows from a stronger result, described in the report on 1.33.

2.26 The functional equation

$$f^n + g^n + h^n = 1$$

cannot have non-constant meromorphic solutions for $n \geq 9$. Gundersen constructed examples of transcendental meromorphic solutions for $n = 5$ and 6 in [26] and [27] respectively. Thus only the cases $n = 7$ and 8 remain unsolved.

2.34 For every $\lambda > 1$ Fryntov [21] constructed an entire function f of order λ with the property

$$\limsup_{r\to\infty} \log L(r)/\log M(r) < -1.$$

2.35 Hayman and Kjellberg [29] gave a positive answer by proving that for any non-constant subharmonic function u and $K > 1$ the set $\{z : u(z) + KB(z)\}$, where $B(z) = \max_{|\zeta|=|z|} u(\zeta)$, has no unbounded components. Furthermore, if the set $\{z : u(z) + B(|z|) < 0\}$ has an unbounded component then u has infinite lower order, or else regular growth and mean or minimal type of order λ, where $0 < \lambda < \infty$, or u is linear.

2.38 Drasin [12] constructed an entire function of order 1 with the property $L(r)M(r) \to 0$.

2.52 This is solved completely by A.A. Goldberg [23], who proved the following stronger result: Let f be a meromorphic function of zero order, satisfying

$$\liminf_{r\to\infty} \frac{N(r,0,f) + N(r,\infty,f)}{\log^2 r} \leq \sigma < \infty,$$

then

$$\limsup_{r\to\infty} \frac{\min_\theta |f(re^{i\theta})|}{\max_\theta |f(re^{i\theta})|} \geq C(\sigma),$$

where

$$C(\sigma) = \left(\prod_{n=1}^{\infty} \frac{1 - q^{2n-1}}{1 + q^{2n-1}}\right)^2, \quad \text{where} \quad q = \exp(-1/(4\sigma)),$$

and this estimate is best possible.

A different proof of the original Barry's conjecture was given by Fenton [19].

2.64 This is completely settled now in [7]: if f is a real entire function with the property that $f''f$ has only real zeros, then f belongs to the Laguerre–Pólya class. The proof uses the previous results by Sheil-Small [57] who solved the problem in the case of finite order, and by Levin–Ostrovskii [38].

2.65 Köhler [33] proved that the answer is "yes", for meromorphic functions and $n = 6$. Namely, if f and g are meromorphic functions, such that $f^{(k)}/g^{(k)}$ are entire and without zeros, for $0 \leq k \leq 6$, then f and g satisfy one of the four relations suggested by Hinkkanen. If one makes additional assumptions about growth of f and g, one needs fewer derivatives to achieve the same conclusion [33, 63]. See also [35] for related results.

2.69 For every $\lambda > 1/2$ Langley [36] constructed an entire function of order λ with the property

$$\liminf_{r \to \infty} \frac{T(r,f)}{T(r,f')} > 1.$$

2.70 Some interesting examples related to this problem are contained in [37].

2.71 Gundersen [28] showed that for given $a > 0$ and $b \geq 0$, there exists an infinite sequence of real numbers $\lambda_k \to +\infty$, so that the differential equation

$$f'' + (az^4 + bz^2 - \lambda_k)f = 0$$

has solutions with infinitely many zeros, and all these zeros, except finitely many of them are real. Rossi and Wang [54] proved that if an equation $f'' + P(z)f = 0$, where P is a polynomial, has a solution with infinitely many zeros, all of them real, then the number of real zeros of P must be less than than $\deg P/2 + 1$, counted with multiplicities. Eremenko and Merenkov [18] proved that for every d there exist polynomials P of degree d such that some solution of the equation $f'' + Pf = 0$ has only real zeros. The zero set of such f can be infinite if and only if $d \not\equiv 2 \pmod 4$.

2.72 Brüggeman [10] and Steinmetz [60] independently gave a positive answer, in fact each of them proved a stronger result than conjectured.

2.75 Goldberg and Ostrovskii [24] solved the problem under the following additional assumption on the sequence of the exponents. There exists an entire function $L(\lambda)$ of exponential type, such that $L(\lambda_n) = 0$ for $n = 1, 2, \ldots$, and

$$\lim_{n \to \infty} \lambda_n^{-1} \log(1/|L'(\lambda_n)|) < \infty.$$

If these conditions are satisfied, the sequence (λ_n) is said to have finite index of concentration. If the sequence of exponents of a Dirichlet series f has finite index of concentration, the only possible indicators are $h(f,\theta) = a(\cos^+\theta)^\rho$, where $a > 0$. In this paper they also obtained other results, with weaker conditions on the sequence (λ_k), and studied the lower indicators as well.

2.78 An analog of the Fatou conjecture for the real quadratic family $\{z \mapsto z^2 + c : c \in \mathbf{R}\}$ (instead of the family R_d of all rational functions of degree $d \geq 2$) has been established now in [25] and [42]

2.79 (a) has been established for the real quadratic family by McMullen [46], and extended to the family $\{z \mapsto z^d + c, c \in \mathbf{R}\}$ by Levin and van Strien [40].
(b) is known for the following subclasses of rational functions:
Colet–Eckmann functions whose Julia set in not $\bar{\mathbf{C}}$ [52],
finitely renormalizable quadratic polynomials without neutral irrational cycle [45].

2.83 (a) The answer is 'yes', if f is a polynomial of degree 2 [65].

2.86 (b) Rempe [53] proved that for $E(z) = \exp z + c$, if E has an unbounded Siegel disk U (of any period) then $c \in \partial E^k(U)$ for some k.

2.87 The first question is the same as 2.77 and 2.67 is a special case of this question. All these remain unsolved. The conjecture that iterates in a wandering domain cannot converge to a fixed point follows from the results of Pérez Marco [50, 51].

2.88 (a) It is known now that B is locally connected in the neighborhoods of certain points [32, 44]
(b) Shishikura [58, 59] proved that the boundary of the Mandelbrot set B has Hausdorff dimension 2.

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Guide to problems and references

Entries A-M refer to the list of references given after the tables.

Table 1. *List of problems proposed*

Topic	A	B	C	H	I	L
1	1–23	24–29	30	–	31–36	37–43
2	1–32	33–46	47–57	58–64	65–68	69–90

Table 2. *Comments on problems*

Problem Number	Topic 1	Topic 2	Problem Number	Topic 1	Topic 2
1	B	B	22	B	B,J
2	B	B	23	B,J,K	M
3	J	D,J	24	C	C
4	B,F	B,F	25	C,G	B,K,M
5		B,F	26		J
6	E,K,M	C,M	27	D	
7	B	D,J	28	C,D,J	B,E,F
8	B	B,C,E,G	29	K	B,F
9	B	E,M	30		B
10	B,K	D,J	31		B
11	B,K	M	32		C,G,K
12	D	M	33	M	
13	B	J	34	K	M
14	B	C,G	35		M
15	B		37	M	J
16	C,M	B,C	38		F,M
17	B	B,E,F,J	40		C,J
18	B,C,G,M	C	41		J
19	D,M	J,K,M	42	M	D,J
20	D,E,M	M	43		D
21	C,K	B,M			

Problem Number	Topic 2
44	C,G
52	M
55	D
57	D
58	J
62	J
63	J
64	M
65	M
66	K
69	M
70	M
71	M
72	M
75	M
78	M
79	M
83	M
86	M
87	M
88	M

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