

Math 366, Quiz 2

NAME:

1a (3 points) For the matrix

$$\begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix}$$

write the characteristic equation and find eigenvalues.

1b (3 points) For the same matrix, find two linearly independent eigenvectors.

Solution. The characteristic equation is

$$\lambda^2 - 2\lambda + 2 = 0.$$

Solutions are

$$\lambda_{1,2} = 1 \pm \sqrt{1 - 2} = 1 \pm i.$$

Equations for eigenvectors for $\lambda_1 = 1 + i$ are

$$(2 - i)v_1 - 5v_2 = 0, \quad v_1 - (2 + i)v_2 = 0,$$

They are satisfied if we take $(v_1, v_2) = (2 + i, 1)$. Then an eigenvector corresponding to $\lambda_2 = 1 - i$ is $(2 - i, 1)$.

2. (4 points) Compute the real and imaginary parts:

$$\left(\frac{1+i}{1-i} \right)^{2015}.$$

Solution.

$$\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{2i}{2} = i.$$

Now 2015 has remainder 3 when divided by 4, so $i^{2015} = -i$.