A polynomial equation in infinitely many variables

Let $N$ be a positive integer, $(z_k)$ a sequence of complex numbers tending to zero, and

$$\sum_k z_k^n = 0 \quad \text{for every integer} \quad n > N,$$

where the series are absolutely convergent. Prove that all $z_k = 0$.

This can be generalized in the following way: Consider the system of equations with respect to complex numbers $z_k$ and $w_k$:

$$\sum_k w_k z_k^n = a_n \quad \text{for every integer} \quad n > N.$$ 

If this system has a solution with different $z_k \to 0$, and such that the series converge absolutely, then such solution is unique.

In the case of finitely many variables, this is called the Sylvester–Ramanujan system, see for example,