Let $f$ be a holomorphic function in the unit disc having a simple zero at 0 and two simple 1-points at $a, \overline{a}$, and no other zeros and 1-points.

1. What is $a_0$, the minimal possible $|a|$? What is the extremal function?

It is conjectured that the extremal function has a double 1-point at $c$, and the map

$$f : \{z : |z| < 1, z \neq 0, z \neq c\} \to C\backslash\{0, 1\}$$

is a covering, such that the simple loop around $0, c$ is mapped to a curve $AB^2$, where $A$ is a simple loop around 0 and $B$ is a simple loop around 1. Such function is unique, and $c \approx 0.0505468$.

So $a_0 \leq c$, and it is conjectured that $a_0 = c$.

2. Same question if the 1-points are at $b, -b$.

Let $b_0$ be the minimal $|b|$. V. Blondel offered a prize in 1994 for finding $b_0$. It is true that $b_0 > c$? This time, there is no reasonable conjecture about the extremal function, but it is known that $0.0145 < b_0 < 0.1148$.