Ma 262, Spring 2002
Practice Exam 2

Problem 1 Consider the system $\mathbf{A}\ \mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} 5 & -2 & 0 \\ 0 & 1 & -1 \\ -4 & 1 & 2 \end{pmatrix},$$

and $b = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. Use row-echelon reduction and back substitution to solve the system.

Problem 2 The system

$$\begin{aligned} x_1 - 2x_3 - x_4 &= 0 \\ 3x_1 - 4x_2 + x_3 - 3x_4 &= 1 \\ -x_1 + 4x_2 - 5x_3 + x_4 &= 0 \end{aligned}$$

has

A. No solutions
B. Infinitely many solutions
C. Exactly one solution

Problem 3 Find the rank of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ 3 & 0 & -3 \\ 2 & 0 & 1 \end{pmatrix}.$$

Problem 4 Using the method of variation of parameters, we know that a particular solution of the differential equation

$$y'' - 2y' + 5y = 2\ \tan(2x)$$

is of the form $y_p = e^x\ [u_1\ \cos 2x\ + u_2\ \sin 2x]$. What is u_2'?

Problem 5 Consider the matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 3 \\ 2 & 6 \end{pmatrix}$$

a) Find $\mathbf{A}^{-1}$ using reduced row-echelon reduction.
b) Use a) to solve the system

$$\begin{aligned} -x_1 + 3x_2 &= -4 \\ 2x_1 + 6x_2 &= 3. \end{aligned}$$

Problem 6 Consider the matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 0 \\ 2 & -1 & 1 \\ 6 & 0 & -2 \end{pmatrix}$$

a) Find the adjoint of $\mathbf{A}$, $adj(\mathbf{A})$.
b) Find the inverse of $\mathbf{A}$ by means of the formula

$$\mathbf{A}^{-1} = \frac{1}{det(\mathbf{A})}\ adj(\mathbf{A}).$$

Problem 7
If

$$det\ \mathbf{A} = \begin{pmatrix} 0 & a & 0 \\ 1 & 2 & 3 \\ 4 & 3 & 6 \end{pmatrix} = 18,$$

a) Find a;
b) Compute $\det \mathbf{A}^T$.

Problem 8
If the 2×2 matrices $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ x & y \end{pmatrix}$ and $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ \frac{1}{4} & \frac{13}{4} \end{pmatrix}$ satisfy $B^2 = A$, then the bottom row of B is equal to
A. $(-1, 0)$
B. $(\frac{1}{2}, \frac{3}{2})$
C. $(-\frac{1}{2}, \frac{3}{2})$
D. $(-\frac{1}{2}, -\frac{3}{2})$
E. $(\frac{1}{2}, -\frac{3}{2})$

Problem 9 Consider the three vectors in $\mathbb{R}^3$

$$\mathbf{v_1} = \begin{pmatrix} 0 \\ -2 \\ -1 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{v_3} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}.$$

Prove that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ span $\mathbb{R}^3$.

Problem 10 Consider the vector space $C^\infty(I)$.

(a) Prove that the three functions

$$2, \ (x+2), \ x(1-x)$$

are linearly independent in $C^\infty(I)$.

(b) Decide whether or not the function $f(x) = 1 + x^2 - 2x$ is in the $span\{2, (x+2), x(1-x)\}$.

Problem 11
Determine which of the following subsets S is a subspace of the vector space $\mathbf{V}$. Provide motivation for your answers.
(i) $\mathbf{V} = \mathbb{R}^3$, $S = \{(x, y, z) \in \mathbb{R}^3 \mid 2(x-1) - 3(y+1) + (z+7) = 2\}$.

(ii) $\mathbf{V} = M_{2\times 2}(\mathbb{R})$, $S = \left\{ \mathbf{A} \in M_{2\times 2}(\mathbb{R}) \mid \mathbf{A} = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \right\}$.

(iii) $\mathbf{V} = C^2(I)$, where I is an interval of the line, $S = \{f \in C^2(I) \mid f''(x) + 4f'(x) - 3f(x) = 1\}$.