Abstract

The classical Euler zeta function (1736) is

\[ \zeta(s) = \sum_{n=1}^{\infty} n^{-s}. \]

The series converges uniformly on compact subsets of the half-plane \( \Re s > 1 \) and represents an analytic function which admits analytic continuation to the complex plane with the exception of a simple pole at \( s = 1 \). The Euler product

\[ \zeta(s)^{-1} = \prod (1 - p^{-s}) \]

taken over the primes \( p \) converges uniformly on compact subsets of the half-plane and denies zeros of the zeta function in the half-plane. The Riemann hypothesis (1859) is the conjecture that the function has no zeros in the half-plane \( \Re s > \frac{1}{2} \). The absence of zeros on the line \( \Re s = 1 \) was shown in 1896 independently by Hadamard and de la Vallée Poussin. Estimates in the right half of the critical strip \( 0 < \Re s < 1 \) show that the product converges and denies zeros of the zeta function there. The estimates are obtained in \( p \)-adic Fourier analysis for all primes \( p \). The origin of the estimates lies in properties of Fourier analysis which are the subject of the lecture. (Course 69000 on the Riemann hypothesis meets MWF at 9:30 in University 319).

Refreshments will be served in the Math Library Lounge at 4:00 p.m.