



PURDUE UNIVERSITY

Department of Mathematics Colloquium

Speaker: Professor Michael Holst, Department of Mathematics, University of California, San Diego

Title: "Some New Existence and Approximation Results for the Constraints in the Einstein Equations"

Date: Tuesday, September 23, 2008

Time: 4:30 P.M.

Place: MATH 175

Abstract

There is currently tremendous interest in geometric PDE, due in part to activities such as the geometric flow program used recently to solve the Poincare conjecture, and to the NSF-funded LIGO project involving the detection of gravitational wave emission and propagation involving numerical solution of the Einstein equations. In this lecture, we consider the coupled nonlinear elliptic constraints in the Einstein equations. The constraint equations must be solved numerically to produce initial data for gravitational wave simulations, and to enforce the constraints during dynamical simulations.

In the first part of the lecture, we consider a thirty-five-year-old open question involving existence of solutions to the constraint equations on space-like hyper-surfaces with arbitrarily prescribed mean extrinsic curvature, and we give a partial answer using a priori estimates and a new type of topological fixed-point argument. In the second part of the lecture, we develop some adaptive numerical methods for which we can prove a number of useful results on convergence, optimality, and scalability. Based on the a priori estimates developed in the first part of the lecture, we establish some critical discrete estimates. We then derive error estimates for Galerkin approximations, and describe a class of nonlinear approximation algorithms based on adaptive finite element methods (AFEM). We establish some new AFEM convergence and optimality results for geometric PDE problems with non-monotone nonlinearities such as the Einstein constraints. We finish by illustrating the algorithms with some examples using the Finite Element ToolKit (FETK).

This is joint work with Gabriel Nagy and Gantumur Tsogtgerel. (Physical Review Letters, Vol. 100 (2008), No. 16, pp. 161101.1, and <http://arxiv.org/abs/0712.0798>).

Refreshments will be served in the Math Library Lounge at 4:00 p.m.