Speaker: Professor Gopal Prasad, University of Michigan
Title: “Fake Projective Spaces”
Date: Tuesday, October 5, 2010
Time: 4:30 P.M.
Place: MATH 175

Abstract

In this talk I will give an exposition of the results of my three joint papers with Sai-Kee Yeung.

A fake projective plane is a smooth compact complex surface which is not the complex projective plane but has the same Betti numbers as the complex projective plane. It is known that such a surface is projective algebraic and it is the quotient of the open unit ball in by a cocompact torsion-free arithmetic subgroup of PU(2,1). The Euler-Poincare characteristic of such an arithmetic subgroup is 3.

The first fake projective plane was constructed by David Mumford in 1979 using p-adic uniformization and two more examples were found using a similar method by Ishida and Kato.

An interesting problem in complex algebraic geometry is to determine all fake projective planes and study their geometric (and arithmetic) properties. Using a formula for the covolume of principal arithmetic subgroups which I had found in 1987, and some intricate number theoretic considerations, we have been able to list all arithmetic subgroups of PU(2,1) whose orbifold Euler-Poincare characteristic is at most 3. From this we get an explicit description of twenty eight finite classes of fake projective planes. Using ingenious computer-assisted group theoretic computations, Donald Cartwright and Tim Steger have shown that these twenty eight classes contain exactly one hundred fake projective planes up to biholomorphism.

From our explicit description of fake projective planes, we are able to derive interesting geometric properties of these exotic complex algebraic surfaces. For example, we have shown that for most (but not for all) of them the canonical line bundle is divisible by 3.

We have also determined all arithmetic subgroups of SU(n-1,1) whose orbifold Euler-Poincare characteristic is reciprocal of a positive integer. We have proved that there are no such subgroups unless n = 3 or 5. For n=5 we have enumerated them all. This led to the very first examples of fake (smooth ball quotients of dimension 4 with same Betti numbers as complex). We found that there are four of them. Each of them is a Shimura variety.

Refreshments will be served in the Math Library Lounge at 4:00 P.M.

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