Abstract

By means of certain dispersive PDEs (such as the nonlinear Klein-Gordon equation) we will exhibit a new family of phenomena related to the ground state solitons. These solitons are exponentially unstable, and one can construct stable, unstable, and center(-stable) manifolds associated with these ground states. In terms of these manifolds one can completely characterize the global dynamics of solutions whose energy exceeds that of the ground states by at most a small amount. In particular, we will establish a trichotomy in forward time giving either finite-time blow up, global forward existence and scattering to zero, or global existence and scattering to the ground state as all possibilities. It turns out that all nine sets consisting of all possible combinations of the forward/backward trichotomies arise. This extends the classical Payne-Sattinger picture (from 1975) which gives such a characterization at energies below that of the ground state; in the latter case the aforementioned (un)stable and center manifolds do not arise, since they require larger energy than that of the ground state. Our methods proceed by combining a perturbative analysis near the ground states with a global analysis away from them.

Refreshments will be served in the Math Library Lounge at 4:00 p.m.