

MATH 450

Supplementary Problems for Burnside's Theorem

1. How many flags of six stripes of equal width can be made using the colors red white and blue? Note two flags are "the same" if you look at them from the "front or back". So

$$|R|W|B|R|W|B|$$

and

$$|B|W|R|B|W|R|$$

are equivalent. (Imagine the flag is attached to a flagpole on one side, but the pole is not part of the flag.)

2. Suppose a roulette wheel has 6 sectors of equal size and you want to color the sectors using n colors. Show the number of ways to do this is

$$\frac{1}{6} (2n + 2n^2 + n^3 + n^6).$$

3. Consider an equilateral triangle subdivided into six right triangles by the lines of reflection for its symmetry group, D_3 . (See the figure below.) Call two colorings equivalent if there is a ϕ in D_3 which takes one to the other. Determine an equation for the number of non-equivalent colorings of this figure using four colors.

