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Lifting Actions in Fibrations

by

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§ 1. Introduction

Let X be a CW complex upon which a topological group G acts. Suppose $F \rightarrow E \xrightarrow{p} X$ is a fibration. The lifting problem is the following:

Can we impose a G action on E such that p is equivariant? If we can do so we shall call the fibration a G-fibration and we say that the action of G on X can be lifted to the fibration $F \rightarrow E \xrightarrow{p} X$.

A slightly different problem, which we shall call the lifting up to homotopy problem is the following:

Is $F \rightarrow E \xrightarrow{p} X$ fibre homotopy equivalent to a G -fibration? In other words, given $F \rightarrow E \xrightarrow{p} X$ and a G action on X , is there a fibration $E' \xrightarrow{p'} X$ such that p' is fibre homotopy equivalent to p and there is a G action on E' such that p' is equivariant?

In this paper we give a complete answer to the second problem, the lifting up to homotopy problem. We then use this answer in two ways. First we explore the degree to which the answer is valid for the first question, the lifting

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