Lifting Actions in Fibrations

by

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§ 1. Introduction

Let X be a CW complex upon which a topological group
G acts. Suppose $F \to E \overset{p}{\to} X$ is a fibration. The **lifting
problem** is the following:

Can we impose a G action on E such that p is equivari-
ant? If we can do so we shall call the fibration a G-fibra-
tion and we say that the action of G on X can be **lifted**
to the fibration $F \to E \overset{p}{\to} X$.

A slightly different problem, which we shall call the
lifting up to homotopy problem is the following:

Is $F \to E \overset{p}{\to} X$ fibre homotopy equivalent to a G-fibration?
In otherwords, given $F \to E \overset{p}{\to} X$ and a G action on X, is
there a fibration $E' \overset{p'}{\longrightarrow} X$ such that p' is fibre homotopy
equivalent to p and there is a G action on E' such that
p' is equivariant?

In this paper we give a complete answer to the second
problem, the lifting up to homotopy problem. We then use
this answer in two ways. First we explore the degree to which
the answer is valid for the first question, the lifting