

question. Second we consider special cases in which our answer assumes forms which give concrete solutions to the lifting problem.

Given the action of G on X , there is associated with this action a universal fibre bundle, $X \xrightarrow{i} X_G \xrightarrow{p} B_G$, of fibre bundles with fibre X and structural group G . Here B_G is the classifying space for G and $X_G = E_G \times_G X$, the twisted product of the contractible G -space E_G with X . The answer to the lifting up to homotopy problem is embodied in the following theorem.

Theorem 1. The fibration $E \xrightarrow{p} X$ is fibre homotopy equivalent to a G -fibration if and only if it is fibre homotopy equivalent to the pullback of a fibration over X_G induced by the inclusion $X \xrightarrow{i} X_G$.

Now theorem 1 gives rise to a version of itself for the lifting problem in the case of covering spaces.

Theorem 5. G lifts to a covering $\tilde{X} \xrightarrow{p} X$ if and only if the covering is a pullback of a covering of X_G by $i: X \rightarrow X_G$.

A similar theorem to theorem 1 figures in the principal bundle lifting problem. That problem is closely related to the lifting problem and it goes as follows:

Let $K \rightarrow E \xrightarrow{p} X$ be a principal K -bundle with a G -action on X ; can we impose an action on E such that p is equivariant and such that the action of G on E commutes with the