Theorem 1: *There is a lifting of the action of* G *on* X *to a fibration fibre homotopy equivalent to* $E \xrightarrow{p} X$ *if and only if* $E \xrightarrow{p} X$ *is fibre homotopy equivalent to the pullback by* $i: X \to X_G$ *of a fibration over* X_G.

The remainder of this section will be devoted to a proof of this theorem.

Lemma 2: *If* $E \xrightarrow{p} X$ *is a G-fibration, then it is a pullback by* i *of a fibration over* X_G *up to fibre homotopy equivalence*.

Proof: Since $E \xrightarrow{p} X$ is a G-fibration, there is an action on E such that p is equivariant. Hence p induces a map $\bar{p}$: $E_G x_G E \to E_G x_G X = X_G$. If we knew that $\bar{p}$ were a Hurewitz fibration then $\bar{p}$ would be the required fibration over X_G. We avoid that question and instead consider the following commutative diagram where the horizontal rows are principal G-fibrations arising from the diagonal action of G on $E_G \times X$ and $E_G \times E$.

$$\begin{array}{ccc} E_G \times E & \longrightarrow & E_G x_G E \\ \Big\downarrow 1 \times p & & \Big\downarrow \bar{p} \\ E_G \times X & \longrightarrow & X_G \end{array}$$