that $E \xrightarrow{p} X$ is a pullback of some fibration $Q \xrightarrow{p'} X/G$. Now $E = \{(x,v) \in X \times Q \mid \rho(x) = p'(v)\}$. Now we define the required lifting of G to E as follows: Let $g \in G$ and define $g\colon E \to E$ by $g(x,v) = (g(x),v)$. This is well defined and it is easily seen to be a lifting.

Now we turn to the proof of theorem 1. We assume that $E \xrightarrow{p} X$ is a pullback from some fibration $P \to X_G$ by $i\colon X \to X_G$. Now i factors as

$$i\colon\ X \xrightarrow{*\times 1} E_G \times X \xrightarrow{q} X_G$$

where $q\colon E_G \times X \to X_G$ is the quotient map and $* \in E_G$ is a base point. Consider the pullback $q^*(P) \to E_G \times X$ of $P \to X_G$. Since the diagonal action of G on $E_G \times X$ is a free action, lemma 3 states that $q^*(P) \to E_G \times X$ is fibre homotopy equivalent to a fibration $R \to E_G \times X$ for which the diagonal action lifts. Composing the fibration $R \to E_G \times X$ with the projection $E_G \times X \to X$ gives us a new fibration $R \to X$ and the action of G on R obviously lifts the original action on X.

Now all that remains to do is show that $E \xrightarrow{p} X$ is fibre homotopy equivalent to $R \to X$. But this follows immediately by considering the following diagram of fibre maps:

$$\begin{array}{ccccc}
E & \longrightarrow & R & \xrightarrow{1} & R \\
\downarrow & & \downarrow & & \downarrow \\
X & \xrightarrow{*\times 1} & E_G \times X & \xrightarrow{\text{Proj.}} & X
\end{array}$$