This composition $E \to R$ on the top line covers the identity of X and is a fibre homotopy equivalence by Dold's theorem [5], since when restricted to any fibre, the map $E \to R$ is a homotopy equivalence from F to $E_G \times F$, the fibre of $R \to X$.

§ 3. The Lifting and Principal Lifting Problems

The main result for the Principal lifting Problem is the Hattori-Yoshida theorem, [11].

Theorem (Hattori-Yosheda): _Let_ $T \to E \overset{p}{\to} X$ _be a principal torus bundle, let_ X _be locally compact and suppose_ G _is a compact Lie group_. _Then_ G _has a principal bundle lifting if and only if_ $E \overset{p}{\to} X$ _is principal bundle equivalent to a pullback of a torus bundle over_ X_G _by_ i.

This result is clearly analogous to theorem 1. Aside from the restrictions on X, and the fibre, note that the Hattori-Yoshida theorem's "Principal bundle equivalence" is replaced by "Fibre homotopy equivalence." We will say that theorem 1 is a Hattori-Yoshida theorem for the lifting up to homotopy problem for any group G and fibration $E \to X$. We say the theorem above is a Hattori-Yoshida theorem for principal bundle liftings on torus bundles where G is a