Suppose that $f\colon E \to E'$ is a fibre homotopy equivalence and let $h\colon E' \to E$ be its fibre homotopy inverse. Now we define the action of G on E by letting $g\colon E' \to E'$ correspond to $fgh\colon E \to E$. We must only check that this is well defined, that is that $f(gg')h = (fgh)(fg'h)$. Now $(fgh)(fg'h) = fg(hf)g'h$ and $hf\colon E' \to E'$ is homotopic to the identity by a homotopy which lies over tne identity on X. Hence $(fgh)(fg'h)$ is homotopic to $fgg'h\colon E \to E$ by a homotopy lying over the identity on X. But since the fibres of $E \overset{p}{\to} X$ are discrete, this homotopy must be the constant homotopy and so $(fgh)(fg'h) = f(gg')h$.

A similar Hattori-Yoshida type theorem holds for principal fibrations with discrete fibres. In this case the fibre homotopy equivalence would be between $E \overset{p}{\to} X$ and $E_G \times K \to E_G \times E \to X$ where the discrete group K acts on $E_G \times E$ as described in the paragraph following Proposition 4. This action implies that the lifting of G on the original $E \overset{p}{\to} X$ lifts as a principal bundle action. This arguement gives us the following theorem.

Theorem 6: <u>If</u> $\tilde{X} \overset{p}{\to} X$ <u>is a regular covering, there is a principal bundle lifting of</u> G <u>to</u> $\tilde{X}$ <u>if and only if</u> $\tilde{X} \overset{p}{\to} X$ <u>is a pullback by</u> i <u>of a regular covering over</u> X_G.