Now the Hattori-Yoshida theorems for principal bundle liftings hold for T-bundles, for regular coverings, for free actions and of course also for trivial actions. This suggests that there is a Hattori-Yoshida theorem for all principal lifting problems. This is false and a counter example is based on the following theorem, due to Glen Bredon, which classifies the principal lifting problem for transitive actions. Note that if G acts transitively on X, then X may be thought of as a homogeneous space G/H where H is the isotropy subgroup of G and G acts on G/H on the left.

Theorem 7: <u>Suppose</u> G <u>acts transitively on</u> G/H <u>with isotropy subgroup</u> H <u>and suppose</u> $E \xrightarrow{p} G/H$ <u>is a principal</u> K <u>bundle.</u> <u>Then</u> G <u>has a principal bundle lifting if and only if the classifying map</u>, k, <u>of</u> $E \xrightarrow{p} G/H$ <u>factors up to homotopy as</u>

$$k\colon G/H \xrightarrow{i} B_H \xrightarrow{B\phi} B_K$$

<u>where</u> $\phi\colon H \to K$ <u>is some homomorphism</u>.

Proof: Suppose there is a principal bundle lifting of G to E. Then G x K acts transitively on E with an isotropy subgroup isomorphic to H. Hence E is homeomorphic to (G x K)/H where H is the subgroup of G x K given