by $h \to h \times \phi(h)$ where $\phi\colon H \to K$ is some homomorphism. Now consider the diagram

$$\begin{array}{ccccccc}
K & & K & & K & & K \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
(G\times K)/H & \longrightarrow & (E_G\times K)/H & \longleftarrow & (E_H\times K)/H & \longrightarrow & E_K \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
G/H & \xrightarrow{\;i\;} & B_H & \xleftarrow{\;\alpha\;} & B_H & \xrightarrow{\;B\phi\;} & B_K
\end{array}$$

where H acts diagonally on $E_G \times K$ and $E_H \times K$. Now $\alpha\colon E_H/H \to E_G/H$ is a homotopy equivalence so we will not count it in our notation.

Conversely, if $E \overset{p}{\to} G/H$ has $k\colon G/H \overset{i}{\to} B_H \xrightarrow{B\phi} B_K$ has a classifying map, the diagram above reveals that $E \overset{p}{\to} G/H$ is principal bundle equivalent to $(G\times K)/H \to G/H$ which admits a principal bundle lifting.

With the aid of the above theorem, we shall see Hattori-Yoshida theorems do not hold for principal bundle liftings for those structure groups K such that some map $B_G \to B_K$ is not induced by a homomorphism $G \to K$. For a Hattori-Yoshida theorem would say there would be a lifting if the bundle classified by $k\colon G/H \to B_K$ factors through $G/H \overset{i}{\to} (G/H)_G = B_H \to B_K$ whereas the theorem above states in addition that the map $B_H \to B_G$ must be induced by a homomorphism $H \to K$. Now B_H is a homogeneous space, E_H/H,