and in this case i is a homotopy equivalence. Thus there is a counterexample to a Hattori-Yoshida theorem whenever there are maps $f: B_H \to B_K$ which are not induced by homomorphisms.

According to D. Sullivan [19, p. 5.95, remark] there is a map $\phi: B_{S^3} \to B_{S^3}$ which is not induced by a homomorphism of S^3 to itself.

For a counterexample in which G is a compact Lie group, we have the following due to Glen Bredon: We let $G = Sp_2$. Then by [19, p. 5.93, corollary 5.10] there is a map $B_{Sp_1} \overset{\phi}{\to} B_{Sp_1}$ which has degree 9 on $H^4(B_{Sp_1})$ for some n. Now we consider the composition

$$S^7 = Sp_2/Sp_1 \overset{i}{\to} B_{Sp_1} \overset{\phi}{\to} B_{Sp}$$

and let $\phi \in \pi_7(B_{Sp_1}) \cong \pi_6(S^3) \cong Z_{12}$ represent the homotopy class of $i: S^7 \to B_{Sp_1}$. Now α is the generator of $\pi_7(B_{Sp_1})$ by the exact sequence arising from the fibration

$$S^7 \overset{i}{\to} B_{Sp_1} \to B_{Sp_2}$$

and the fact that $\pi_7(B_{Sp_2}) = 0$. Now $\phi_*(\alpha) = 9\,\alpha \in \pi_7(B_{Sp_1})$ since ϕ^* for degree 9 on $H^4(B_{Sp_1})$. But ϕ could be replaced by a homomorphism only if $\phi_*(\alpha)$ were either 0 or $\pm\alpha$ in $\pi_7(B_{Sp_1}) \cong Z_{12}$, since if a map from $S^3 \to S^3$ were a homomorphism it would have degree 0 or $\pm$ 1.