

There still remains the possibility that a Hattori-Yoshida theorem is true for a broad class of lifting problems, as opposed to principal bundle lifting problems. However in this case also there is a counter example is based on the method of the previous counterexample and upon the following lemma which relates the lifting problem to the principal bundle lifting problem.

Lemma 8: If the fibre bundle $F \rightarrow E \rightarrow X$ admits a lifting of G , then the associated principal $\mathcal{A}(F)$ -bundle admits a principal bundle lifting.

By $\mathcal{A}(F)$ we mean the group of homeomorphisms of the fibre F . If $E \xrightarrow{p} X$ is an oriented bundle, in the sense that $\pi_1(X)$ acts trivially on the components of $\mathcal{A}(F)$, then $\mathcal{A}(F)$ can be taken to be the identity component of itself.

Proof: Let $E^{(F)}$ denote the space of homeomorphisms of F onto fibres of E . Then $\mathcal{A}(F) \rightarrow E^{(F)} \rightarrow X$ is the associated principal bundle to $E \rightarrow X$. The lifting of G on E gives rise to a principal bundle lifting of G on $E^{(F)}$ by $g: E \rightarrow E$ inducing $g_{\#}: E^{(F)} \rightarrow E^{(F)}$ where $g_{\#}(f) = g \circ f$ where $f \in E^{(F)}$.

Hence to find a counterexample to a Hattori-Yoshida type theorem for fibre bundles, we must find a space $F = G/H$ and