a map $\phi: B_H \to B_{\mathscr{A}(F)}$ such that

$$G/H \overset{i}{\to} B_H \overset{\phi}{\to} B_{\mathscr{A}(F)}$$

is not homotopic to a map where ϕ is replaced a map induced by a homomorphism $H \to \mathscr{A}(F)$.

Let $F = S^1 = S^3/S^1$.

Consider $\mathscr{A}(S^2)$, the group of homomorphisms of S^2 which are isotopic to the identity. By Kneser [14], we know that $SO(3)$ is a strong deformation retract of $\mathscr{A}(S^2)$. If $\bar{h}$: $SO_3 \to \mathscr{A}(S^2)$ is the inclusion homomorphism, the induced map on classifying spaces h: $BSO_3 \to B_{\mathscr{A}(S^2)}$ is a homotopy equivalence.

Now let $S^3 \overset{\bar{\alpha}}{\to} SO_3$ be the double covering, so $\bar{\alpha}$ induces a double covering $B_{S^3} \overset{\alpha}{\to} B_{SO_3}$.

Let $\phi: B_{S^3} \to B_{S^3}$ be the map from before which does not induce a homomorphism from $S^3 \to S^3$.

Now S^3 acts on S^2 in only two ways up to conjugation by a homeomorphism of S^2; (1) the trivial action, or (2) the action given by the coset representation of $S^2 = S^3/S^1$. This can be seen by inspecting the subgroups of S^3 to see which can give rise to isotropy subgroups. Only S^3 and S^1 are possible isotropy subgroups.

Now since there are so few actions of S^3 on S^2 there must be few homomorphisms of S^3 into $\mathscr{A}(S^2)$. In