compare theorem 1 and the Hattori-Yoshida theorem to show that the lifting up to homotopy problem and the principal bundle lifting problem are equivalent for torus bundles, at least when G is connected.

Proposition 9: *Suppose* $E \xrightarrow{p} X$ *is a principal torus bundle,* X *is locally compact and* G *is a compact connected Lie group. Then* $E \xrightarrow{p} X$ *admits a principal bundle lifting if and only if there is a lifting up to homotopy.*

Proof: We only need show that if G lifts up to homotopy, there exists a principal bundle lifting. By theorem 1, we know there exists a fibration $T \to E' \to X_G$ which pulls back to a fibration which is fibre homotopy equivalent to $E \xrightarrow{p} X$. First we shall show that $E' \to X_G$ is an "oriented" fibration, by which we mean that $\pi_1(X_G)$ acts as the trivial group of homotopy equivalences on the fibre T. Then we shall show that an oriented fibration with fibre a torus is fibre homotopy equivalent to a principal torus bundle, in fact that the fibre homotopy classes of oriented fibrations with fibre T is in one to one correspondence with Principal bundle equivalence classes of Principal torus bundles. Hence we can apply the Hattori-Yoshida theorem.

Now G is connected, so $\pi_1(B_G) = 0$. Hence $\pi_1(X) \xrightarrow{i_*} \pi_1(X_G)$ is onto. Since $\pi_1(X)$ acts trivially on T because $E \to X$ is a principal bundle, it follows that