

$\pi_1(X_G)$ acts trivially on T . Hence $E' \rightarrow X_G$ is oriented.

Now by [7, see p. 54] we know that the classifying space B_∞ for fibrations with fibre a torus has only two nonzero homotopy groups, $\pi_1(B_\infty)$ and $\pi_2(B_\infty) \cong \mathbb{Z}^n$ where n is the dimension of T . Now $E' \rightarrow X_G$ is classified by a map $k': X_G \rightarrow B_G$. Now k_* is trivial on $\pi_1(X_G) \rightarrow \pi_1(B_\infty)$ since $E' \rightarrow X_G$ is oriented. Thus k' factors through the universal covering \tilde{B}_∞ which is $K(\mathbb{Z}^n, 2)$, the classifying space for principal T bundles. Hence we can see the one to one correspondence between principal T bundles and oriented fibrations with fibre T .

One final remark about the proof of the Hattori-Yoshida theorem for principal T -bundles. Hattori and Yoshida's proof does not follow from theorem 1. It employs the theory of group cohomology with continuous cochains. For Principal S^1 -bundles, or equivalently, complex line bundles, there are alternative proofs. One proof reportedly exists in a partial manuscript by Graeme Segal. This proof uses Segal's modification of the cohomology of groups with continuous cochains. An alternative method, told to me by Peter Landweber, uses equivariant K -theory and the Atiyah-Segal completion theorem [1] to characterize the group of G -equivariant complex line bundles (with tensor product as the group multiplication) over X as $H^2(X_G, \mathbb{Z})$. Then those