We may rephrase the problem by asking the following question: Given a fibration $F \to E \to B$, under what circumstances is there a covering of E, namely $\tilde{E}$, such that $\tilde{E} \to B$ has fibre $\tilde{F}$, a specified covering of F? Thus for $X = F$ and $X_G = E$, this covering question is equivalent to the pullback question and hence to the lifting problem.

The covering problem for universal coverings is completely solved in [2, see theorem 1]. Combining theorem 5 with this result we have

Theorem 5': Let $\tilde{X} \to X$ be a universal covering. Then G lifts if and only if the following two conditions are satisfied:

a) $i_*: \pi_1(X) \to \pi_1(X_G)$ is injective ,

b) $p_*: \pi_1(X_G) \to \pi_1(B_G)$ has a right inverse (which is a homomorphism).

Shortly after [2] appeared, R. Stong sent me a group theoretic solution for any covering space, not just universal coverings. Combining that result with theorem 5 we get

Theorem 5": The covering $\tilde{X} \to X$ admits a lifting of G if an only if the following two conditions are satisfied where the covering corrosponds to the subgroup $A \subset \pi_1(X)$, N is the normalizer of the image of i_* in $\pi_1(X_G)$, and