Conversely, we suppose that $\hat{\omega}^*(k) = 1 \times k$. Now the diagram gives rise to a mapping ϕ^{**} of spectral sequences, $\phi^{**} = E^{**} \to \bar{E}^{**}$ where $\bar{E}^{**}$ denotes the spectral sequence of $G \times X \to E_G \times X \to B_G$. We must show that for $k \in H^2(X) = E_2^{0,2}$ we have $d_2(k) = 0$ and $d_3(k) = 0$. Now observe that $1 \times k \in H^2(G \times X) = \bar{E}^{0,2}$ is in the image of $(i \times 1)^*$, hence $\bar{d}_2(1 \times k) = 0$ and $\bar{d}_3(1 \times k) = 0$. Now on the E_2 level, $\phi_2 = 1 \otimes \hat{\omega}^*$. Also observe that $\hat{\omega}^*$ must be injective. Thus $\phi_2^{2,1}$ is injective. Since $\phi_2^{2,1}(d_2(k)) = \bar{d}_2 \phi_2^{0,2}(k) = \bar{d}_2(1 \times k) = 0$, we see that $d_2(k) = 0$. Now consider $d_3(k) \in E_3^{0,3}$. Since G is connected, $H_1(B_G) = 0$ so $E_3^{3,0} = \bar{E}_3^{3,0} = H^3(B_G)$ and $\phi_3^{3,0}$ is the identity. Thus $d_3(k) = \phi_3^{3,0}(d_3(k)) = \bar{d}_3(\phi_3^{0,2}(k) = \bar{d}_3(1 \times k) = 0$. Hence k is in the image of ϕ^*.

Proposition 15: Let $k \in H^2(X,\pi)$ classify the oriented fibration $K(\pi,1) \to E \to X$ and let G be connected. Then G lifts up to homotopy if and only if $\hat{\omega}^*(k) = 1 \times k$.

By Proposition 9 we immediately apply Proposition 15 to obtain the solution for Principal bundle lifting problems for Principal T-bundles. Assume that G is connected and