$T \to E \to X$ is classified by $k \in H^2(X;Z^n)$ where X is a locally compact CW complex.

Theorem 16: <u>There is a principal bundle lifting for compact connected Lie groups</u> G <u>if and only if</u> $\hat{\omega}^*(k) = 1 \times k$.

For the remainder of this section we will develop consequences of theorem 16. The equation

$$\hat{\omega}^*(k) = (1 \times k) + \omega_1 + (\omega^*(k) \times 1)$$

gives two obstructions to lifting G: namely $\omega_1 \in H^1(X;\ H^1(X;Z^n))$ and $\omega^*(k) \in H^2(G;\ Z^n)$. Now $\omega_1 = 0$ whenever $H^1(X;H^1(G;Z^n)) = 0$, that is whenever $H_1(X;Z)$ or $H_1(G;Z)$ is torsion. The second obstruction $\omega^*(k)$ is frequently zero. For example:

1. $\omega^* = 0$ if the action has a fixed point.
2. $\chi(X)\omega^* = 0$ if X is compact, [3, theorem 1.1]
3. $\omega^* = 0$ if $H_1(G;Z)$ is free and $H_1(X;Z)$ is torsion, [9].

To see why 3 is true we first note that $\omega_*: H_2(G;Q) \to H_2(X;Q)$ is trivial by using the fact that $H_1(X;Q)$ is zero. Since $H_1(G;Z)$ is free, that implies that $H^2(G;Z^n)$ is free by the universal coefficient theorem. Hence $\omega^*: H^2(X;Z^n) \to H^2(G;Z^n)$ must be trivial.

The conditions of 3 also imply that $\omega_1 = 0$, hence we have: