Corollary 17: There always exists a principal T-bundle lifting when $H_1(G;Z)$ is free and $H_1(X;Z)$ is torsion for X locally compact and G a compact connected Lie group.

Comparing Theorem 16 with previous results on the lifting problem we see that it embraces Stewart's theorem [17] that there always is a lifting for G simply connected, and J. E. Su's theorem [18] that S^1 can always be lifted if $H_1(X;Z)$ is torsion.

As an example of our techniques, we finish this section by studying the principal S^1-bundle $S^1 \to S^{2n-1} \to CP^n$ and applying theorem 16. Since CP^n is simply connected, $\omega_1 = 0$. Hence the only obstruction to lifting is $\omega^*(k)$ where k is a generator of $H^2(CP^n;Z) \cong Z$. Now any $U(k)$ action on CP^n must lift since $\pi_1(U(k)) \cong Z$ and so corollary 17 applies. On the other hand, the cononical action of the projective unitary group $PU(n)$ on CP^n does not lift since $\omega^*(k)$ is a nonzero element of order $n + 1$.

Now let us consider $SO(k)$ actions on CP^n. If n is even, then $\chi(CP^n) = n + 1$ is odd. Since $\pi_1(SO(k)) \cong Z_2$ (if $k > 2$) we have $H^2(SO(k);Z) \cong Z_2$. Then the fact that $0 = \chi(CP^n)\omega^* = (n + 1)\omega^*$ and $n + 1$ is odd implies that $\omega^*(k) = 0$. Hence we see that every $SO(k)$ action lifts on CP^{2n}. This is not the case for CP^{2n+1} of course. For example, the cononical action of $SO(3)$ on $CP^1 = S^2$ does