not lift to S^3 in the Hopf fibration $S^1 \to S^3 \to S^2$ since, as is well known, $\omega^*(k) \neq 0$

Now we employ an arguement due to R. Schultz to use information about liftings to discover facts about ω^*

Proposition 18. <u>Suppose</u> $SO(k)$ <u>acts effectively on</u> CP^n. <u>Then</u> $\omega^* = 0: H^2(CP^n;Z) \to H^2(SO(k);Z)$ <u>if</u>

a) $n < 2^{r-1}$ <u>for</u> $k \not\equiv 0 \pmod 4$ <u>and</u> $r = [\frac{k}{2}]$,

or

b) $n < 2^{r-2} + r$ <u>for</u> $k \equiv 0 \pmod 4$ <u>and</u> $r = \frac{k}{2}$.

Proof: We prove that $SO(k)$ actions must lift to $S^1 \to S^{2n-1} \to CP^n$ under the above hypothesis by means of a theorem of W. Y. Hsiang, hence the obstruction to lifting $\omega^*(k) = 0$ and since k is the generator of $H^2(CP^n;Z)$, we will be done. Now W. Y. Hsiang's theorem [13, theorem V.9] states that if $Spin(k)$ acts effectively on R^n, then $n \geq 2^r$ if $k \not\equiv 0 \pmod 4$ and $n \geq 2^{r-1} + 2r$ if $k \equiv 0 \pmod 4$ where $k = [k/r]$. Now if $Spin(k)$ acts effectively on a sphere S^n, we can "cone" the action to get an effective action of $Spin(k)$ on R^{n+1}. Hence Hsiang's theorem states that an effective $Spin(k)$ action on S^n implies that $n \geq 2^r - 1$ if $k \not\equiv 0 \pmod 4$ and $n \geq 2^{r-1} + 2r - 1$ if $k \equiv 0 \pmod 4$. Then effective $Spin(k)$ actions on S^{2n-1} imply that $n \geq 2^{r-1}$ if $k \not\equiv 0 \pmod 4$ and $n \geq 2^{r-2} + r$ if $k \equiv 0 \pmod 4$.