we regard $\langle e,x \rangle_G \equiv \langle e\cdot g^{-1}, g(x) \rangle_G$. Now the inclusion map $i: X \to X_G$ given by $i(x) = \langle 1,x \rangle_G$, where $1 \in E_G$ is the identity element, is equivariant. So the cononical G action on X_G extends the action of G on X.

Proposition 20: <u>Every map into</u> X_G <u>with the cononical action of</u> G <u>can be made equivariant up to homotopy</u>.

Proof: In view of theorem 1, we must show that every fibration over X_G is the pull back of some fibration over $(X_G)_G$ by the inclusion map $i: X_G \to (X_G)_G$. This follows since $(X_G)_G = B_G \times X_G$, which is the following lemma.

<u>Lemma 21</u>: $(X_G)_G = B_G \times X_G$.

Proof: We regard an arbitrary element in $(X_G)_G$ as having the form $\langle e,e',x \rangle_G$ where

$$\langle eg^{-1}, ge'h^{-1}, hx \rangle_G \equiv \langle e,e',x \rangle_G$$

for arbitrary $g,h \in G$. Here $e, e' \in E_G$. On the other hand we regard an arbitrary element of $B_G \times X_G$ as having the form $\langle e \rangle_G \times \langle e',x \rangle_G$ where

$$\langle eg^{-1} \rangle_G \times \langle e'h^{-1}, hx \rangle_G \equiv \langle e \rangle_G \times \langle e',x \rangle_G$$

for arbitrary $g,h \in G$.